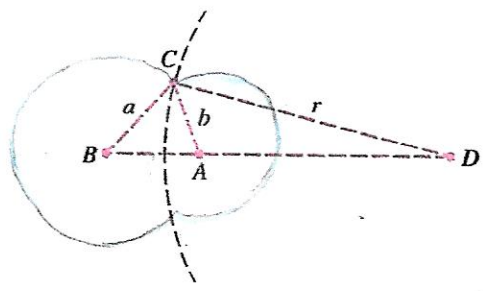


Math 196L – 001 (Fall 2017)

**Instructions:** Read each problem. Write a sentence or two about the approach you might take to solve each problem. Draw a picture to illustrate the scenario. Write a formula that might be needed to help set up or solve the problem.

[1] **SOAP BUBBLES** When two bubbles cling together in midair, their common surface is part of a sphere whose center  $D$  lies on the line passing through the centers of the bubbles (see figure below). Also, angles  $ACB$  and  $ACD$  each have measure  $60^\circ$

- (a) Show that the radius  $r$  of the common face is given by  $r = \frac{ab}{a-b}$ .  
(Hint: Use the Law of Sines together with the fact that an angle  $\theta$  and its supplement  $180^\circ - \theta$  have the same sine.
- (b) Find the radius of the common face if the radii of the bubbles are 4 cm and 3 cm.
- (c) What shape does the common face take if the two bubbles have equal radii?



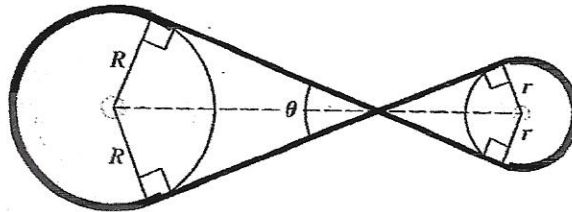
[2] **BELTS AND PULLEYS** A thin belt of length  $L$  surrounds two pulleys of radii  $R$  and  $r$ , as shown in the figure below.

(a) Show that the angle  $\theta$  (in radians), where the belt crosses itself, satisfies the equation

$$\theta + 2 \cot\left(\frac{\theta}{2}\right) = \frac{L}{R+r} - \pi$$

[Hint: Express  $L$  in terms of  $R$ ,  $r$ , and  $\theta$  by adding up the lengths of the curved and straight parts of the belt.]

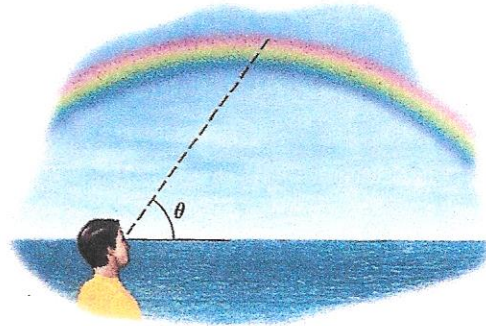
(b) Suppose that  $R = 2.42 \text{ ft}$ ,  $r = 1.21 \text{ ft}$ , and  $L = 27.78 \text{ ft}$ . Find  $\theta$  by solving the equation in part (a) graphically. Express your answer in radians.



[3] **RAINBOWS** → Rainbows are created when sunlight of different wavelengths (colors) is refracted and reflected in raindrops. The angle of elevation  $\theta$  of a rainbow is always the same. It can be shown that  $\theta = 4\beta - 2\alpha$ , where

$$\sin \alpha = k \sin \beta$$

and  $\alpha = 59.4^\circ$  and  $k = 1.33$  is the index of refraction of water. Use the given information to find the angle of elevation  $\theta$  of a rainbow.



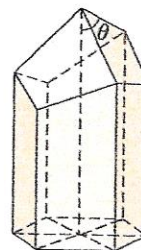
- [4] **BEEHIVES** → In a beehive each cell is a regular hexagonal prism, as shown in the figure. The amount of wax  $W$  in the cell depends on the apex angle  $\theta$  and is given by

$$W = 3.02 - 0.38 \cot \theta + 0.65 \csc \theta$$

Bees instinctively choose  $\theta$  so as to use the least amount of wax possible.

- (a) Use a graphing device to graph  $W$  as a function of  $\theta$  for  $0 < \theta < \pi$ .
- (b) For what value of  $\theta$  does  $W$  have its minimum value?

[Note: Biologists have discovered that bees rarely deviate from this value by more than a degree or two.]



### EXAMPLE 5 | Navigation

A pilot sets out from an airport and heads in the direction  $N 20^\circ E$ , flying at 200 mi/h. After one hour, he makes a course correction and heads in the direction  $N 40^\circ E$ . Half an hour after that, engine trouble forces him to make an emergency landing.

- Find the distance between the airport and his final landing point.
- Find the bearing from the airport to his final landing point.

